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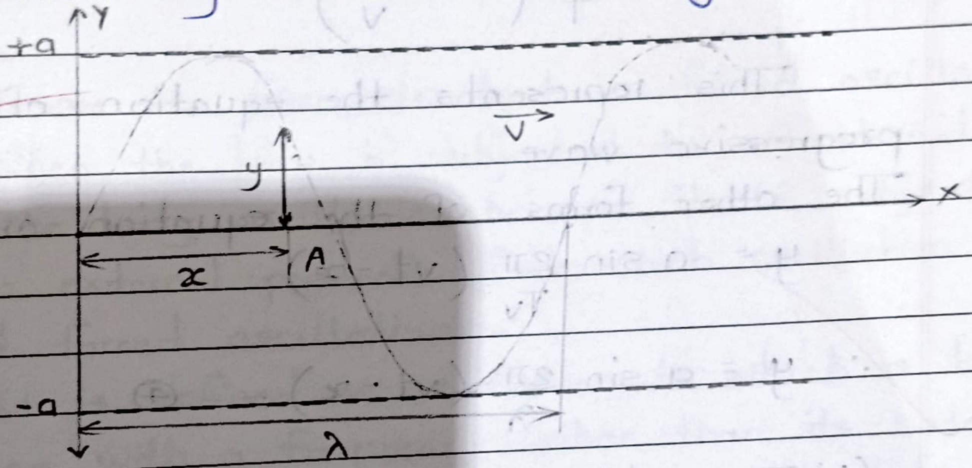
Q.1) Obtain the differential equation of wave function.

→ A wave motion is a disturbance which travels through available space or medium and the medium particles vibrates around their mean position when the wave approaches. The motion is handed over through one particle to the next after regular interval of time.

Consider a progressive wave originating at the origin O and travelling along the position x-axis with evelocity v as shown in fig.



Progressive wave

As the wave proceeds, each successive particle of the medium is set into simple harmonic motion. let time be measured from the instant when the particle at the origin 'O' is passing through its equillibrium position. The displacement y of a particle at O from its mean position at any time t is given by,

$$y = a \sin \omega t \quad \text{--- (1)}$$

$$\therefore y = a \sin \frac{2\pi}{T} t \quad \text{--- (2)}$$

Where, $\omega = \frac{2\pi}{T}$

Now consider a particle at point a (A) at a distance x from O. the wave starting from O would reach the point on in seconds later (x/v) than the particle at O. Therefore, there is a phase lag of (x/v) sec between the particle at points A and O. Therefore the displacement of the particle at point A at a time will be same as that of particle at O at a time (x/v) sec earlier ie. at time $(t - x/v)$. Thus eq^n becomes

$$y = a \sin \frac{2\pi}{T}\left(t - \frac{x}{v}\right) \quad \text{--- (3)}$$

This represents the equation of a plane progressive wave.

The other forms of the equation are,

$$y = a \sin \frac{2\pi}{Tv}(vt - x)$$

$$\therefore \; y = a \sin \frac{2\pi}{\lambda}(vt - x) \quad \text{--- (4)}$$

$(\because Tv = v/f = f\lambda/f = \lambda)$

Also,

$$y = a \sin 2\pi\left(\frac{t}{T} - \frac{x}{vT}\right)$$

$$\therefore \; y = a \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) \quad \text{--- (5)}$$

For diff eq^n of a wave diff. eq^n (4) w.r.t x

$$\frac{dy}{dx} = a\left(\frac{2\pi}{\lambda}\right) \cos \frac{2\pi}{\lambda}(vt - x)$$

$$\text{or } \frac{d^2y}{dx^2} = -a\left(\frac{2\pi}{\lambda}\right)^2 \sin \frac{2\pi}{\lambda}(vt - x) \quad \text{--- (6)}$$

Again diff eq^n (4) w.r.t t we get,

$$\frac{dy}{dt} = a\left(\frac{2\pi}{\lambda}\right) v \cos \frac{2\pi}{\lambda}(vt - x)$$

$$\text{or } \frac{d^2y}{dt^2} = -a\left(\frac{2\pi}{\lambda}\right)^2 v^2 \sin \frac{2\pi}{\lambda}(vt - x) \quad \text{--- (7)}$$

Comparing eq^n (6) and (7)

$$\frac{d^2y}{dx^2} = \frac{1}{v^2}\frac{d^2y}{dt^2} \quad \text{--- (8)}$$

$$\text{or, } \frac{d^2y}{dt^2} = v^2\frac{d^2y}{dx^2} \quad \text{--- (9)}$$

2) Which are the forces involved in forced oscillation.

→ When the body is subjected to an external force here the body oscillates, because it is subjected to an external periodic force such oscillation is called forced oscillation.

It is defined as oscillation in which a body vibrates with a frequency other than its natural frequency under the action of an external periodic force.

In this, following forces are acting on the body.

i) Restoring force $= -ky$

ii) Resisting force $= -r\frac{dy}{dt}$

iii) External periodic force $= F\sin pt$

$$ma = -ky + r\frac{dy}{dt} + F\sin pt$$

$$\therefore m\frac{d^2y}{dt^2} + r\frac{dy}{dt} + ky = F\sin pt$$

$$\therefore \frac{d^2y}{dt^2} + \frac{r}{m}\frac{dy}{dt} + \frac{k}{m}y = \frac{F}{m}\sin pt$$

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Where, $\frac{r}{m} = 2b$; $\frac{k}{m} = \omega^2$; $\frac{F}{m} = f$

$$\therefore \frac{d^2y}{dt^2} + 2b\frac{dy}{dt} + \omega^2 y = f\sin pt \quad \text{--- (I)}$$

--- [diff eq^n of forced oscillation].

At steady state body oscillates with the frequency of applied force and not with its natural frequency.

The sol^n of the above eq^n is.

$$y = A\sin(pt - \theta)$$

Where,

A = amplitude

P = frequency of force or derived frequency.

θ = Phase of oscillation

ω = natural frequency of body

$$\therefore \frac{dy}{dt} = AP\cos(pt - \theta)$$

$$\text{or } \frac{d^2y}{dt^2} = -AP^2\sin(pt - \theta)$$

$\therefore$ Eq^n (I) becomes.

$$\therefore -AP^2\sin(pt-\theta) + 2bAP\cos(pt-\theta) + \omega^2 A\sin(pt-\theta) = f\sin pt$$

$$[\text{i.e. ; RHS} = f\sin pt = f\sin[(pt-\theta)+\theta]];$$

$$\because f\sin(pt-\theta)\cos\theta + f\cos(pt-\theta)\sin\theta$$

$$\therefore A(\omega^2 - P^2)\sin(pt-\theta) + 2AbP\cos(pt-\theta) =$$

$$= f\sin(pt-\theta)\cos\theta + f\cos(pt-\theta)\sin\theta$$

Now,

Comparing coeff. of $\sin(pt-\theta)$ & $\cos(pt-\theta)$ of both

$$\therefore A(\omega^2 - P^2) = f\cos\theta \quad \text{--- (I)} \quad \&$$

$$2AbP = f\sin\theta \quad \text{--- (II)}.$$

Squaring and adding eq^n I and II

$$A^2(\omega^2-p^2)^2 + 4A^2b^2p^2 = f^2(\cos^2\theta + \sin^2\theta)$$

$$A^2(\omega^2-p^2)^2 + 4A^2b^2p^2 = f^2$$

$$\therefore A^2\left[(\omega^2-p^2) + 4b^2p^2\right] = f^2$$

$$\therefore A^2 = \frac{F^2}{(\omega^2-p^2)+4b^2p^2}$$

$$\therefore A = \frac{F}{\sqrt{(\omega^2-p^2)+4b^2p^2}} \quad \text{--- [Amplitude].}$$

Divide eq^n ~~I by II~~ by I

$$\frac{2AbP}{A(\omega^2-p^2)} = \frac{F\sin\theta}{F\cos\theta} = \tan\theta$$

$$\therefore \theta = \tan^{-1}\left(\frac{2bP}{\omega^2-p^2}\right) \quad \text{--- [Phase of oscillation.]}$$

Q.3) What is free vibration oscillation. Derive an expression for differential eq^n of free vibration oscillation.

→ Consider the motion of a particle of mass m on which a restoring force is acting, such that particle perform harmonic oscillation. For harmonic oscillation the restoring force is linearly proportional to the displacement i.e. $-ky$ where K is restoring force constant and negative sign indicates that it acts in opposite direction to displacement.

i.e. $F = -ky$ --- (1)

Acc^{dn} to Newton's law.

$$F = ma.$$

i.e. $F = m\frac{d^2y}{dt^2}$ --- (2)

Comparing eq^n (1) and (2)

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$$m\frac{d^2y}{dt^2} = -ky$$

i.e. $\frac{d^2y}{dt^2} + \frac{k}{m}y = 0.$

Let, $\frac{k}{m} = \omega^2$

$\therefore \frac{d^2y}{dt^2} + \omega^2 y = 0$ ---- (3)

The solⁿ of above eqⁿ is given in the form.

$y = e^{\alpha t}$

$\therefore \frac{dy}{dt} = \alpha e^{\alpha t}$

& $\frac{d^2y}{dt^2} = \alpha^2 e^{\alpha t}$ ---- (4)

Substituting above values in eqⁿ (3).

$\alpha^2 e^{\alpha t} + \omega^2 e^{\alpha t} = 0$

i.e. $(\alpha^2 + \omega^2) e^{\alpha t} = 0$

As, $e^{\alpha t} \neq 0$

$\therefore \alpha^2 + \omega^2 = 0$

$\therefore \alpha^2 = -\omega^2$

$\therefore \alpha = \pm i\omega$

Thus, the general solⁿ of eqⁿ (3) is given by

$y = Ae^{i\omega t} + Be^{-i\omega t}$

Where, A & B are constant to be determined.

$\therefore y = A(\cos\omega t + i\sin\omega t) + B(\cos\omega t - i\sin\omega t)$

$y = (A+B)\cos\omega t + i(A-B)\sin\omega t$

Let, $A + B = R\sin\phi$

and $i(A-B) = R\cos\phi$

$y = R\sin\phi\cos\omega t + R\cos\phi\sin\omega t$

$y = R\sin(\omega t + \phi)$ ---- (5)

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From eq^n (5) it is clear that R is the maximum value of y. Thus R is the amplitude of oscillation. The value of y repeats when t changes by $2\pi/\omega$ i.e.

$$y = R\sin[\omega(t + 2\pi/\omega) + \phi]$$
$$y = R\sin[(\omega t + \phi) + 2\pi]$$
$$y = R\sin(\omega t + \phi)$$
$$y = y$$

Thus after time interval of $2\pi/\omega$, the motion will repeat itself. The interval $2\pi/\omega$ is called the periodic time T.

$$\therefore T = 2\pi/\omega.$$

The frequency,

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

$$\therefore F = \frac{1}{2\pi}\sqrt{\left(\frac{k}{m}\right)} \quad \text{--- (6)}$$

From above eq^n it is clear that the amplitude of oscillation is constant or independent of time.

Q.4) Which are the forces acting in the damped oscillation? Obtain the differential equation of damped oscillation.

→ The time required to die out the oscillation will depend on the magnitude of the resistive force. A motion damped by resistive force results Damped oscillation.

The resistive force is proportional to the veloci[ty] and in the direction opp. to direction of motion. A damped system has following forces:-

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i) Restoring force :- $-ky$

ii) Resistive force :- $-r\frac{dy}{dt}$.

Where r is frictional force per unit velocity

The above forces are balanced by Newton's f...

i.e $ma = -ky - r\frac{dy}{dt}$ ... (1)

$\therefore m\frac{d^2y}{dt^2} + r\frac{dy}{dt} + ky = 0$

$\frac{d^2y}{dt^2} + \frac{r}{m}\frac{dy}{dt} + \frac{k}{m}y = 0$.

Let $\frac{r}{m} = 2b$ and $\frac{k}{m} = \omega^2$

$\therefore \frac{d^2y}{dt^2} + 2b\frac{dy}{dt} + \omega^2 y = 0$ ... (2)

The sol^n of eq^n (2) will be in form

$y = Ae^{\alpha t}$ — (3)

Where, A is orbitary constant.

Diff. eq^n (3)

$\frac{dy}{dt} = A\alpha e^{\alpha t}$

and $\frac{d^2y}{dt^2} = A\alpha^2 e^{\alpha t}$.

$\therefore$ eq^n (2) becomes.

$A\alpha^2 e^{\alpha t} + 2bA\alpha e^{\alpha t} + \omega^2 Ae^{\alpha t} = 0$.

$\therefore Ae^{\alpha t}(\alpha^2 + 2b\alpha + \omega^2) = 0$.

But $Ae^{\alpha t} \neq 0$.

$\therefore \alpha^2 + 2b\alpha + \omega^2 = 0$, ... (4)

$\therefore$ Roots of eq^n (4) gives.

$\alpha = \frac{-2b \pm \sqrt{(4b^2 - 4\omega^2)}}{2}$

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$\therefore \alpha = -b \pm \sqrt{(b^2 - \omega^2)}$ --- (5)

The general sol^n of eq^n (2) is given by,

$$y = Ae^{(-b+\sqrt{(b^2-\omega^2)})t} + Be^{(-b-\sqrt{(b^2-\omega^2)})t}$$

Q.5) Derive the eq^n for Under damped oscillation.
→ When $b^2 < \omega^2$, the components $\sqrt{(b^2-\omega^2)}$ is imaginary.

The sol^n will be given by,

$$y = Ae^{(-b+i\beta)t} + Be^{(-b-i\beta)t}$$

Where, $\beta = \sqrt{(b^2-\omega^2)}$

and $i = \sqrt{(-1)}$

$$y = e^{-bt}\left(Ae^{i\beta t} + Be^{-i\beta t}\right)$$

$$y = e^{-bt}\left[A(\cos\beta t + i\sin\beta t)\right] + \left[B(\cos\beta t - i\sin\beta t)\right]$$

$$y = e^{-bt}\left[(A+B)\cos\beta t + i(A-B)\sin\beta t\right]$$

Let, $A+B = a\sin\phi$

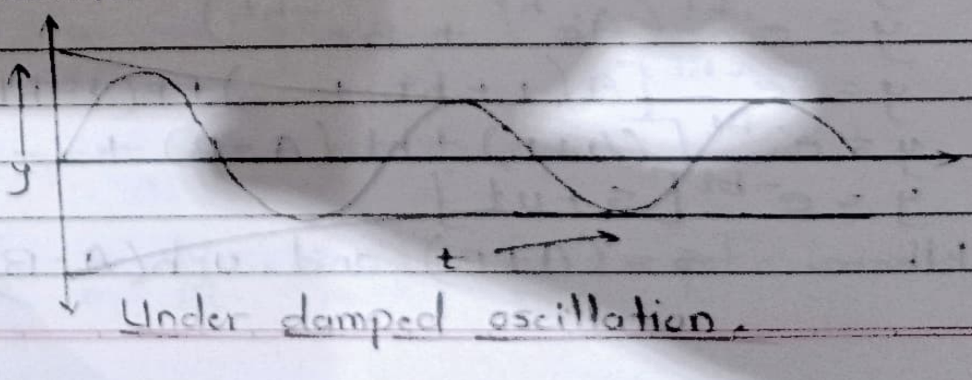
& $i(A-B) = a\cos\phi$

$$\therefore y = e^{-bt}(a\sin\phi\cos\beta t + a\cos\phi\sin\beta t)$$

$$y = e^{-bt}a\sin(\beta t + \phi)$$

The eq^n represents the simple harmonic motion with amplitude ae^{-bt}. The amplitude of motion will continuously decrease because of the factor e^{-bt}. The factor e^{-bt} is called the damping factor and b the damping coeff.

Example of under damped oscillation is pendulum in air, electric oscillator.



Under damped oscillation.

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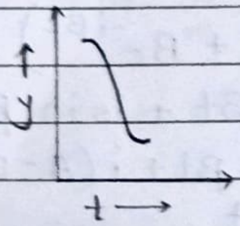
Q. 6) What are the different types of Damped oscillat^n explain them with example.

→ 1) Over damped or Dead Beat

When $b^2 > \omega^2$, $\sqrt{(b^2-\omega^2)}$ is real and less than b. In this case power of both exponents is negative. Thus the displacement y consists of two terms both are decreasing exponentially. This type of motion is called Over damped or Dead beat

• Eg:- pendulum in thick oil



Over Damped oscillation.

2) Critically Damped :-

When $b^2 = \omega^2$, if we put $b^2 = \omega^2$ in the sol^n it will not satisfy the diff. eq^n.

∴ assume that $\sqrt{(b^2-\omega^2)}$ is not zero but is tending to zero i.e equal to very small quantity h.

∴ sol^n becomes

$$y = Ae^{(-b+h)t} + Be^{(-b-h)t}$$

$$y = e^{-bt}(Ae^{ht} + Be^{-ht})$$

$$y = e^{-bt}[A(1+ht+ \ldots) + B(1-ht+ \ldots)]$$

$$y = e^{-bt}[(A+B) + ht(A-B) + \ldots]$$

$$y = e^{-bt}[s+ut]$$

Where, $s = (A+B)$ and $u = h(A-B)$

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As t increases the factor e^{-bt} decreases and $[s + ut]$ increases $\therefore$ the displacement approaches zero as t increases. Such a motion is called critically damped.

- Eg :- pointer of voltmeter & current meter which comes to rest without oscillation.

Q.7) What is forced oscillation? Derive formula for amplitude in case of forced oscillation.

→ Body oscillates because it is subjected to an external periodic force. such oscillation is called forced oscillation.

A forced oscillation can be defined as the oscillat^n in which a body ~~vibrates~~ with a frequency other than its natural frequency under the action of an external perodic force.

In a forced oscillation, the force acting on the body are :-

i) Restoring force :- $-kx$.

ii) Resistive force :- $-r\frac{dy}{dt}$

iii) External periodic force, f ~~sip~~ $\sin pt$

The sum of above eq^n is balanced by Newton's force i.e. $ma = -ky - r\frac{dy}{dt} + f\sin pt$.

$$\therefore m\frac{d^2y}{dt^2} = -ky - r\frac{dy}{dt} + f\sin pt$$

$$\therefore m\frac{d^2y}{dt} + r\frac{dy}{dt} + ky = f\sin pt$$

$$\therefore \frac{d^2y}{dt^2} + \frac{r}{m}\frac{dy}{dt} + \frac{k}{m}y = \frac{f}{m}\sin pt$$

Taking $\frac{r}{m} = 2b$, $\frac{k}{m} = \omega^2$, $\frac{f}{m} = f$.

$$\therefore \frac{d^2y}{dt^2} + 2b\frac{dy}{dt} + \omega^2 y = f\sin pt \quad \text{--- (a)}$$

— [Diff eq^n of motion of particle].

At steady state the body oscillates with frequency of applied force and not with its natural frequency. The sol^n of eq^n will be form; as,

$$y = A\sin(pt-\theta)$$

Where A is arbitary const. Now diff. eq^n.

$$\therefore \frac{dy}{dt} = Ap\cos(pt-\theta)$$

$$\text{and } \frac{d^2y}{dt^2} = -Ap^2\sin(pt-\theta)$$

Substitute in (a) eq^n.

$$\therefore -Ap^2\sin(pt-\theta) + 2bAp\cos(pt-\theta) + \omega^2 A\sin(pt-\theta)$$
$$= f\sin pt = f\sin[(pt-\theta)+\theta]$$

$$\therefore A(\omega^2-p^2)\sin(pt-\theta) + 2bAp\cos(pt-\theta)$$
$$= f\sin(pt-\theta)\cos\theta + f\cos(pt-\theta)\sin\theta.$$

Campairing coeff. of $\sin(pt-\theta)$ and $\cos(pt-\theta)$ on both side, we get.

$$A(\omega^2-p^2) = f\cos\theta.$$
$$2bAp = f\sin\theta.$$

Sqquaring & Adding above eq^n.

$$A^2(\omega^2-p^2)^2 + 4b^2A^2p^2 = f^2(\cos^2\theta+\sin^2\theta)$$
$$A^2[(\omega^2-p^2)^2 + 4b^2p^2] = f^2.$$
$$\therefore A = \frac{f}{\sqrt{[(\omega^2-p^2)+4b^2p^2]}}$$

Now, dividing these eq^{ns}.

$$\therefore \tan\theta = \frac{2bAp}{A(\omega^2 - p^2)}$$

$$\therefore \theta = \tan^{-1}\frac{2bAp}{A(\omega^2 - p^2)}$$

Depending upon relative values of p and ω we have following cases:-

- Case 1 :- When driving frequency is low i.e. $p << \omega$,

$$A = \frac{f}{\sqrt{[(\omega^2 - p^2)^2 + 4b^2p^2]}}$$

$$= \frac{f}{\sqrt{\omega^4}} = \frac{f}{\omega^2} = \text{const}$$

$$\& \quad \theta = \tan^{-1}\left(\frac{2bp}{\omega^2 - p^2}\right) = \tan^{-1}(0) = 0.$$

- Case 2 :- When $p = \omega$

$$\therefore A = \frac{f}{2bp} = \frac{f}{r\omega}$$

$$\& \quad \theta = \tan^{-1}\left(\frac{bp}{0}\right) = \tan^{-1}(\infty) = \pi/2$$

Displacement & force will have phase diff of $\pi/2$.

- Case 3 :- When $p >> \omega$

$$\therefore A = \frac{f}{\sqrt{p^4 + 4b^2p^2}} = \frac{f}{p^2} = \frac{f}{mp^2}.$$

$$\& \quad \theta = \tan^{-1}\left(\frac{2bp}{\omega^2 - p^2}\right) = \tan^{-1}\left(\frac{2bp}{-p^2}\right)$$

$$= \tan^{-1}(0) = \pi$$

∴ Displacement & force will have phase diff of π.

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Q. 8) What is resonance? What are the factors which decides the sharpness of resonance?

→ A body vibrates with the frequency of the external force causing the oscillation rather than its natural frequency. The resultant amplitude under ~~standing~~ forced vibration is given as

$$A = \frac{f}{\sqrt{(\omega^2-p^2)+4b^2p^2}} \quad \text{— (I)}$$

From above eq^n it is clear that the resultant amplitude of oscillation varies with the frequency value of force p. For a particular value of p the amplitude becomes maximum. This phenomenon is known as Resonance.

Thus, a phenomenon of making a body oscillates with is natural frequency under the influence of another oscillating body with the same frequency is called resonance. For ex.

amplitude to be maximum, i.e.

$\sqrt{(\omega^2-p^2)^2+4b^2p^2}$ has to be maximum

i.e. $\frac{d}{dp}\left[(\omega^2-p^2)^2+4b^2p^2\right]=0.$

$\therefore (\omega^2-p^2)(-2p)+4b^2(2p)=0.$

$\omega^2-p^2=2b^2.$

or $p=\sqrt{(\omega^2-2b^2)}$.

if the damping is small i.e. b is negligible then above eq^n becomes to:

$p=\omega.$

Which is the condit^n for resonance.

Substituting this cond^n in eq^n (I)

$$\therefore A_{max} = \frac{f}{\sqrt{(p^2-p^2)^2+4b^2p^2}} = \frac{f}{2bp}.$$

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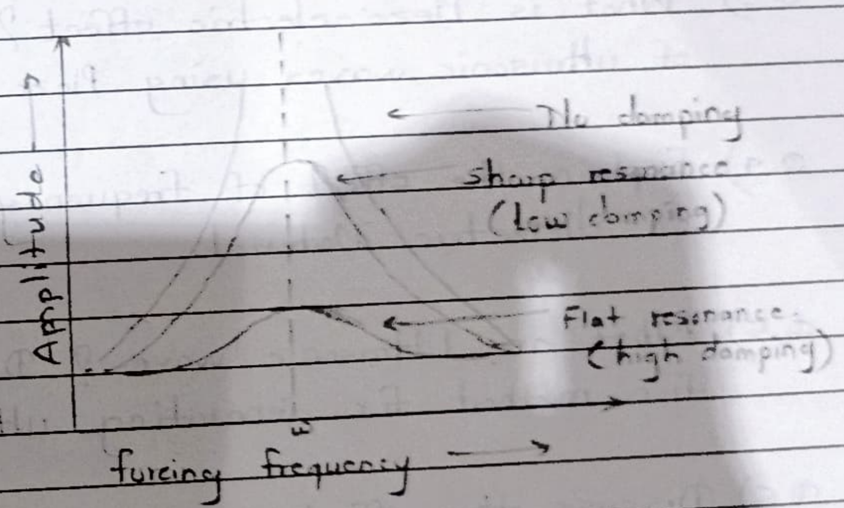
Thus A_{max} approaches to infinity when damping force approaches to zero.

• Sharpness of Resonance :-

1) The amplitude of forced oscillation is maximum when the frequency of the applied force satisfies the condⁿ of resonance i.e $p = \sqrt{(\omega^2 - 2b^2)}$ If the frequency changes from these values the amplitude falls. The rate of fall in amplitude with the change of forcing frequency on each side of the resonance frequency is called sharpness of resonance.



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Q.1) What is Piezo-electric and magnetostriction effect.

Q.1) What is natural frequency of 40mm length of a pure iron rod. Given density of iron is 7.25×10^3 kg/m^2 and it's Young's modulus is 115×10^9 N/m^2. Can you use it in magnetostriction oscillator to produce ultrasonic waves.

Q.2) Explain the terms :-
i) Dielectric constant.
ii) Electric displacement
iii) Polarizability.

Q.3) What is Piezo-electric effect ? Explain production of ultrasonic waves using Piezo-electric oscillator.

Q.4) Explain the effect of frequency and temperature on dielectric Material.

Q.5) What are Ultrasonic waves ? Discribe magnetostriction method for generating ultrasonic waves.

Q.6) Discuss the effect of temperature and frequency dependence on polarization in dielectrics.

Q.7) What is constructive & Distructive interference.

Q.8) What are Newton's rings ? Explain How they are formed.

Q.9) Derive an expression for the optical path difference for the reflected rays in a thin film of constant thickness & Hence find the conditions

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For maxima and minima.

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